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On the limits to speculation in centralized versus decentralized market regimes ☆

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Abstract

Speculation creates an adverse selection cost for utility traders, who will choose not to trade if this cost exceeds the benefits of using the asset market. However, if they do not participate, the market collapses, since private information alone is not sufficient to create a motive for trade. There is, therefore, a limit to the number of speculative transactions that a given market can support. This paper compares this limit in decentralized, monopoly-intermediated and competitively-intermediated market regimes, finding that the second regime is best equipped to deal with speculation: an informed monopolist can price-discriminate investors and thus always avoid market breakdowns. These regimes are also compared in terms of welfare and trading volume. The analysis suggests a reason for the presence of intermediaries in financial markets.

JEL classification: D84; G10

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1. Introduction

If speculation, or information-based trading, is to be profitable, it must be at the expense of regular traders or investors (henceforth *utility traders*.) Utility traders use asset markets for purposes unrelated to asset-specific payoff information, usually categorized as

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consumption smoothing, insurance (hedging), and so on. However, these utility traders will choose not to participate if the adverse selection costs imposed by the actions of informed speculators exceed the benefits of using the market.

Nevertheless, as is widely known, the market needs utility traders in order to operate, since private information alone is not sufficient to create trade. A market composed solely of speculators is characterized by a null volume of transactions (no-trade theorem). It follows that there is a limit to the number of speculative transactions, compared to non-speculative transactions that a given market can afford. This limit is determined by the maximum rents that can be extracted from utility traders before they abandon the market. Since one would expect maximal rent extraction to depend on the organizational form of the market, the limit on speculative transactions can also be expected to vary with organizational form.

This leads me to the question I confront in this paper: how does the functioning of a secondary asset market vary across different organizational forms:

(1) a disintermediated, direct search market, where buyers and sellers are randomly matched;
(2) an intermediated market, where a monopolistic broker-dealer concentrates all transactions; and
(3) an intermediated market, where a set of broker-dealers compete for orders?

The cases of informed and uninformed intermediaries are distinguished.

The main goal is to understand how the limit on speculative transactions varies across these market regimes. An improved understanding of this issue can further illuminate the link between market viability (robustness) and organizational form. My main finding is that an informed, monopolistic market maker is better equipped to deal with the adverse selection created by speculation, as she can keep the market open no matter how strong adverse selection is. The reason is that such an intermediary may offer *favorable selection* to her uninformed clients, by rejecting trades that are favorable to her. Such a strategy has the advantage of allowing the monopolist to raise transaction fees, to a level that extracts all informational rents from speculators, thereby neutralizing their destructive effect on the market.

The idea that adverse selection may lead to a market breakdown is as old as the idea of adverse selection itself (Akerlof, 1970). Milgrom and Stokey (1982) brought it into the analysis of information-based trading in financial markets, and proved that a market where it is commonly known that all traders are speculators cannot exist—the so-called "no-trade theorem."

Glosten (1989) shows that a monopolistic intermediary can prevent market breakdowns in situations where a competitive group of intermediaries would not. The mechanism by which this works is, however, radically different from the one studied here. Glosten shows that an uninformed intermediary may prefer to engage in a negative expected value trade, in the hope of recouping the loss in the future. Prospects improve with trading because the monopolist learns from trades, reducing the informational disadvantage, and consequently the adverse selection cost, while maintaining the possibility of making profits on liquidity traders.

In contrast, in this paper the informed monopolist price discriminates speculators and utility traders by exploiting his information in the form of favorable selection. This can be implemented by a strategy that combines brokerage with good advice.

This strategy, however, can only be implemented if the intermediary can build a reputation that renders the promise of good advice credible. This reduces the likelihood of the same strategy being implemented by a set of competitive intermediaries. Moreover, even if such a strategy were credible, the forces of competition would not permit it, because a competitor could always find another profitable strategy in response.

Bhattacharya and Spiegel (1991) also consider an informed monopolist who sells to risk-averse uninformed buyers. Besides the informational motive, the monopolist has a hedging motive for trading, which prevents prices from being fully revealing. The market breaks down when the informational motive is too strong compared to the hedging motive. In contrast to the present setting, the monopolist is assumed to be incapable of credibly committing to any strategy, much less offering favorable selection.

A related issue, which I do not address, is that of endogenous information production. One could ask, for instance, what kind of market structures favor research by investors (e.g. Grossman and Stiglitz, 1980). Instead, I treat information as exogenously given.

Besides the possibility of market breakdowns, I also compare these regimes in terms of trading volume and welfare. Centralized markets are thought to be better in general because they produce better matches (Garbade, 1982); this is partially reflected in my analysis, as trading volume is generally higher than in the decentralized regime. Total welfare is, consequently, higher in centralized markets, because utility traders have guaranteed access to trade, provided a breakdown does not occur.

The existence of the adverse selection cost, born by the uninformed when they trade with the informed, suggests a reason for the existence of intermediaries in financial markets, unrelated to the incentive problems advanced by Leland and Pyle (1977). In a decentralized market that suffers significantly from this 'lemon problem,' one would expect to see a spontaneous move towards intermediation. A broker-dealer who announces a taking-all-bets policy will attract utility traders and hence concentrate trade. She could also collect fees, which utility traders will pay to avoid adverse selection and speculators will later pay to trade at all. The fact that she can collect fees from both types of trader implies that her existence is always viable. Hence, adverse selection is a force favoring intermediation in financial markets. See Allen (1990) for an approach to intermediary existence related to this and Bhattacharya and Thakor (1993) for a comprehensive review of this literature.

To address these issues, I use a random matching model in which players are paired to voluntarily bet on the occurrence of two states. Trade is modeled by the simultaneous acceptance of a bet. By modeling a betting game rather than a game in which players actually trade an asset, I hope to simplify the analysis, while capturing its essence. The key observation is that ultimately, any decision to buy or sell an asset involves a bet: the buyer is betting that the price will not drop the next day; the seller is betting on the opposite. Regardless of the particular reasons for buying or selling an asset, the decision to do it today rather than tomorrow is predicated on a particular belief about today's price compared to tomorrow's.

The rest of this paper is organized as follows: Section 2 introduces the model. Section 3 is devoted to the analysis of the decentralized market, while Section 4 studies the market with an intermediary. Section 5 analyzes the possibility of competition among intermediaries. Section 6 compares the three regimes in terms of welfare and trading volume. Section 7 concludes.

2. The model: a betting game

There is a continuum of risk-neutral players with common priors, with Lebesgue measure 1 on the interval $[0, 1]$. Half are assigned the role of "buyer," the other half the role of "seller." There is nothing to buy or sell: the name of "buyer" or "seller" is purely metaphorical.

When the game starts, every buyer is randomly matched to a seller, and vice versa. Then, speculators get to see a signal $\omega \in \{\omega_1, \omega_2\}$, while utility traders see nothing.[1] At this point, everyone is offered a bet: buyers are offered the possibility of winning \$1 if state θ_1 occurs or losing \$1 if θ_2 occurs. Sellers are offered the complementary bet, that is, the possibility of losing \$1 if state θ_1 occurs or winning \$1 if θ_2 occurs.

In each match, the bet is carried out only when both players accept. If any player rejects the bet, the game ends and both players get a utility of 0. On the other hand, if they both accept, everybody gets to see the state $\theta \in \{\theta_1, \theta_2\}$, and the payments are made.

Speculators and utility traders differ not only in the information they obtain, but also in their preferences. In particular, utility traders enjoy gambling, getting a utility level of $x > 0$ just for betting, on top of any expected gain from the bet. This is a way of reflecting any non-informational motive for trading, such as liquidity needs, consumption smoothing, and the like. It implies that a utility trader would be willing to accept even actuarially unfair bets to participate in the market, just as a liquidity-shocked person is willing to incur up to a certain loss to sell quickly. Speculators, in contrast, participate only if they expect to win.

The modeling choice of a unique x rather than one for each utility trader is not consequence-free: it simplifies computations, but excludes some interesting features of standard comparisons of competitive and monopolistic markets. This issue will be discussed in Section 6.

Thus, if $a_i \in \{\text{Accept the bet, Reject the bet}\} \equiv \{1, 0\}$ is player i's decision, while $a_{-i} \in \{1, 0\}$ his opponent's decision, the ex post payoff to each player $i \in [0, 1]$ is given by

$$u_i(a_i, a_{-i} \mid \theta) = \begin{cases} x + 1 & \text{if } a_i a_{-i} = 1,\ \theta = \theta_1 \text{ and } i \text{ is a Utility Trader,} \\ x - 1 & \text{if } a_i a_{-i} = 1,\ \theta = \theta_2 \text{ and } i \text{ is a Utility Trader,} \\ 1 & \text{if } a_i a_{-i} = 1,\ \theta = \theta_1 \text{ and } i \text{ is a Speculator,} \\ -1 & \text{if } a_i a_{-i} = 1,\ \theta = \theta_2 \text{ and } i \text{ is a Speculator,} \\ 0 & \text{for any type of player if } a_i a_{-i} = 0. \end{cases} \tag{1}$$

[1] Note that we could have defined four types, instead of two. We omitted the informed individuals who enjoy gambling and the uninformed individuals who do not enjoy gambling. This exclusion was deliberately made for simplicity. Besides, it comes at no cost, as these types would play no role, given that they would behave the same as the two remaining types.

Table 1
Prior beliefs over payoff-relevant and informational events

	ω_1	ω_2
θ_1	$(1+g)/4$	$(1-g)/4$
θ_2	$(1-g)/4$	$(1+g)/4$

The bet is ex ante actuarially fair. That is, the prior probability of θ_1 is 0.5. It is further assumed that each signal, ω_1 and ω_2, is equally likely. Thus, the joint prior probability over (ω, θ) is parametrized as shown in Table 1.

From the table we can obtain speculators' beliefs upon receiving each signal:

$$\Pr(\theta_1 \mid \omega_1) = \frac{(1+g)/4}{(1+g)/4 + (1-g)/4} = \frac{1}{2}(1+g) = \Pr(\theta_2 \mid \omega_2), \tag{2a}$$

$$\Pr(\theta_1 \mid \omega_2) = \frac{(1-g)/4}{(1-g)/4 + (1+g)/4} = \frac{1}{2}(1-g) = \Pr(\theta_2 \mid \omega_1). \tag{2b}$$

Note that a speculator cannot learn anything from the behavior of her opponent, since she knows at least as much as he does. This is not true in the case of a utility trader, so in general it will be necessary to also know equilibrium strategies to compute his beliefs.

If $(1+g)/4 > 1/4$, or equivalently, if $g > 0$ as it is assumed, the message $\{\omega_1\}$ indicates that state θ_1 is the most likely (and hence is an invitation to buy), and so is $\{\omega_2\}$ for θ_2. Consequently, a buyer who receives the signal ω_1 would think of the bet as actuarially favorable to him, and unfavorable in the case of ω_2.

Note that parameter g is the expected gain for a buyer, conditional on receiving a signal ω_1 (the symmetry imposed on the joint distribution of (ω, θ) implies that g is also the expected gain of a seller conditional on receiving a signal ω_2, since $\Pr(\theta_2 \mid \omega_2) = \Pr(\theta_1 \mid \omega_1)$). Therefore,

$$g = (1)\Pr(\theta_1 \mid \omega_1) + (-1)\Pr(\theta_2 \mid \omega_1) = (1)\Pr(\theta_2 \mid \omega_2) + (-1)\Pr(\theta_1 \mid \omega_2). \tag{3}$$

Let z denote the measure of utility traders in the total population. Table 2 presents the mass of each trader type in the population.

Throughout we will assume that $x < g$; otherwise, the utility from gambling would be so high compared to the expected monetary gain/loss that a utility trader would not care about the possibility of entering an actuarially unfair gamble. In reality, although this is certainly not reflected in the static model of this paper, one is inclined to believe that no matter how willing the buyer is to pay for an asset, he would postpone the transaction if he

Table 2
The distribution of types

	Buyer	Seller	TOTAL
Speculator	$(1-z)/2$	$(1-z)/2$	$(1-z)$
Utility trader	$z/2$	$z/2$	z
TOTAL	$1/2$	$1/2$	

Table 3
Expected utilities of accepting the bet

Message	Role	Expected utility
$\{\omega_1\}$	Buyer	$g\,\Pr(\text{opponent accepts})$
$\{\omega_1\}$	Seller	$-g\,\Pr(\text{opponent accepts})$
$\{\omega_2\}$	Buyer	$-g\,\Pr(\text{opponent accepts})$
$\{\omega_2\}$	Seller	$g\,\Pr(\text{opponent accepts})$
$\{\omega_1, \omega_2\}$	Buyer	$\{(x+g)/2\,\Pr(\text{opponent accepts if } \omega_1)$ $+ (x-g)/2\,\Pr(\text{opponent accepts if } \omega_2)\}$
$\{\omega_1, \omega_2\}$	Seller	$\{(x-g)/2\,\Pr(\text{opponent accepts if } \omega_1)$ $+ (x+g)/2\,\Pr(\text{opponent accepts if } \omega_2)\}$

thinks it is very likely that the price will be significantly lower the next day. The assumption of x being smaller than g, then, could be considered as a metaphor for the rational timing of the transaction decision.

It follows that a speculator in possession of good news will always accept, while one in possession of bad news never will: there is nothing else that this person could learn, either by direct observation or by inferring from other people's behavior, that would make him change his mind.[2] He simply knows whether the gamble is fair or unfair to him.

Thus, the problem is in the hands of utility traders. Their non-participation would lead to no trade, in which case the market would break down. However, they will participate (that is, they will accept the bet when offered) as long as the expected monetary loss due to the participation of speculators does not outweigh the utility from gambling, x. Table 3 reports the expected utilities of each type, conditional on the behavior of the opponents.

We can verify in Table 3 that, for a speculator, the expected utility from trading is either proportional to g or to $-g$, so that the decision is unambiguous, as claimed earlier. However, this is not true in the case of a utility trader; I analyze her decision in the next section.

3. Decentralized equilibrium

Consider the case of a utility trader in the role of buyer. There are two possibilities: either she is matched to another (uninformed) utility trader, in which case she faces a fair gamble that is worth accepting, because she gets x; or, she is matched to a speculator, who would in turn accept the bet only if the odds of winning are against the buyer. Hence, her expected utility can be decomposed as:

$$zx + (1-z)\left(\frac{1}{2}(x+g)(0) + \frac{1}{2}(x-g)(1)\right). \tag{4}$$

The first term corresponds to the probability of being matched to a utility trader, a case in which she faces a zero-expected value gamble and gets x. The second term corresponds

[2] This is a consequence of assuming that there is one signal common to all, rather than one for each individual.

Table 4
Optimal decisions

Speculators	$\{\omega_1\}$	$\{\omega_2\}$
Buyer	Accept	Reject
Seller	Reject	Accept
Utility traders	$\{\omega_1, \omega_2\}$	
Buyer	Accept if (5) is met, reject otherwise	
Seller	Accept if (5) is met, reject otherwise	

to the probability of being matched to a speculator, in which case she knows that the trade favorable to her (which would mean a utility of $(x + g)$ to her) would be rejected, while the unfavorable trade ($(x - g) < 0$ to her), would be accepted with probability 1.

As she cannot a priori distinguish between a utility trader and a speculator, she would accept the bet if she thinks that it is likely enough that she is in the first situation and not in the second one. Rearranging terms, Eq. (4) is non-negative if:

$$\frac{1}{2}(x - g) + \frac{1}{2}(x + g)z \geqslant 0, \tag{5a}$$

$$\Leftrightarrow \quad x \geqslant g\frac{1 - z}{1 + z}, \tag{5b}$$

$$\Leftrightarrow \quad z \geqslant \frac{g - x}{g + x}. \tag{5c}$$

The analysis for the seller's acceptance decision is identical. The optimal decision for each type of trader is summarized in Table 4.

Equation (5) is in fact a necessary condition for the existence of the market, for if it is not met, utility traders do not participate. In fact, it is easy to see that in a game between two speculators there can be no trade, because in the case of trade the payoff to the buyer is the negative of the seller's (as Table 3 shows), while not accepting the bet leads to a zero payoff. Put another way, the game between two speculators is a zero-sum game, where all equilibria involve no trade.

From the point of view of a utility trader, the probability of facing an unfair gamble is determined by the proportion of speculators in the population, $(1 - z)$. Depending on how valuable information is, a different level of utility x (gains from trade) will be required in order to support trade for a given composition of the population. The more valuable information is—the bigger g is—the higher the adverse selection problem to the uninformed, and hence the higher the value of trading the asset must be so that she still wants to trade. This can be seen in Eq. (5b). Alternatively, Eq. (5c) shows that for a fixed value of x, to maintain trade while increasing g will require a reduction in the proportion of informed traders $(1 - z)$.

In other words, the cost of the adverse selection problem to a utility trader is determined together by g, the individual information rent, and $(1-z)$, the probability of being matched to a speculator, totaling $g(1 - z)/2$. This cost must be smaller than the utility she gets by gambling, x, with probability $(1 + z)/2$.

It is interesting to note that there is a trade-off between the maximum proportion of speculators in the economy and the predictive power of their information. For instance, if the information service is not very accurate, g is small, and the maximum number of speculators can be very large with respect to the number of utility traders. That is, a very small proportion of the transactions needs to be non-speculative. This appears to be the case in the foreign exchange market, characterized by a huge volume of trade that appears many times larger than needed as a means of exchange, and traders who make a relatively small profit on each transaction.

Another way to look at condition (5a) is that x and z determine the size of the pie, which in the limiting population composition is completely exhausted by speculators, whereas g is the size of the individual portions, i.e., the per-speculator rent. How many of them we can get is a matter of dividing xz by g.

As a summary, Section 3 already proved that:

Theorem 1. *The decentralized market exists if and only if* $x \geqslant g(1-z)/(1+z)$. *Otherwise, there are no transactions.*

4. One intermediary

Notice that in the decentralized equilibrium, utility traders lose to speculators; the only reason they still trade is the probability of being matched to play a fair gamble, which would allow them to gain the utility from gambling, and offset the risk of losing to the better-informed players. Imagine now that one player announces that she will accept all bets, from anyone, no matter what. That single player is telling the uninformed that she will solve their adverse-selection problem, so they will prefer to trade with her and even be willing to pay a transaction fee, rather than trading in the anonymous decentralized market. However, if all the uninformed prefer to trade with her, then the decentralized economy is left only with speculators, making the market disappear as per the no-trade theorem. Thus, she will centralize all trading, since utility traders prefer to trade with her, while speculators are forced to trade with her when they lose the decentralized market.

This section assumes the existence of a monopolistic intermediary, and asks about the maximum speculative activity that a market so configured can afford. It analyzes two cases: the case of an informed intermediary who could offer adverse or favorable selection to her clients, and the case of an uninformed intermediary who is constrained to use the same policy regardless of the realized state of nature.

4.1. Informed monopoly

An informed monopolistic intermediary may accept or reject a bet from any single player who communicates his intention of trading with her. As bettors are anonymous, except for their roles, this variable takes the form of a probability of accepting a bet from each of them, possibly depending on whether she faces a buyer or a seller, and on the message received. A second choice variable is the transaction fee.

Earlier it was mentioned that the transaction fee is the only way the intermediary has to collect profits. The reason is that if she tries to profit from her private information by awarding higher probabilities of acceptance in the cases in which the public is at a disadvantage, she is reintroducing the adverse-selection problem the utility traders are trying to avoid. To attract them, she must offer better conditions than the decentralized market. In profiting from private information, she collects money *only* from utility traders, while by charging a transaction fee she is also able to extract money from speculators, thereby increasing total revenue.

Let y_ω^r be the proportion of bets accepted from role r players ($r = b$ if buyer, $r = s$ if seller) after receiving a signal ω, and let c denote the transaction fee. The problem for this monopolistic intermediary is to maximize the following objective:

$$\frac{c}{2}\left[\left(\frac{y_{\omega_1}^b}{2} + \frac{zy_{\omega_1}^s}{2}\right) + \left(\frac{y_{\omega_2}^s}{2} + \frac{zy_{\omega_2}^b}{2}\right)\right] + \frac{g}{2}\left[\left(\frac{zy_{\omega_1}^s}{2} - \frac{y_{\omega_1}^b}{2}\right) + \left(\frac{zy_{\omega_2}^b}{2} - \frac{y_{\omega_2}^s}{2}\right)\right]. \tag{6}$$

Expected profit therefore consists of the transaction fee that is collected from all buyers and just utility traders among sellers if the information is ω_1, or from all sellers and just utility traders among buyers, if ω_2; plus, the expected payoff formed by the gap between buyers and sellers on each ω, all weighted by the probability of a buyer or seller accepting, in each state.

In maximizing this objective, the monopolist is constrained by the participation of utility traders and speculators, that is,

$$\frac{1}{2}(x + g - c)y_{\omega_1}^b + \frac{1}{2}(x - g - c)y_{\omega_2}^b \geqslant 0, \tag{7a}$$

$$\frac{1}{2}(x - g - c)y_{\omega_1}^s + \frac{1}{2}(x + g - c)y_{\omega_2}^s \geqslant 0, \tag{7b}$$

$$c \leqslant g. \tag{7c}$$

Equation (7a) establishes that a buyer utility trader will be willing to trade when the monopolist would accept with probability $y_{\omega_1}^b$ in state ω_1, and with probability $y_{\omega_2}^b$ in state ω_2. Equation (7b) is the corresponding one for a seller utility trader. Equation (7c) is the participation condition for speculators: a speculator only wants to trade when there is an expected gain g, and will trade if the transaction fee does not completely consume this gain.

These participation constraints can be rewritten as

$$c \leqslant x + g\left(\frac{y_{\omega_1}^b - y_{\omega_2}^b}{y_{\omega_1}^b + y_{\omega_2}^b}\right), \qquad c \leqslant x + g\left(\frac{y_{\omega_2}^s - y_{\omega_1}^s}{y_{\omega_1}^s + y_{\omega_2}^s}\right) \quad \text{and} \quad c \leqslant g. \tag{8}$$

We can further simplify the problem by exploiting the symmetry between speculators on each side, as well as utility traders on each side. Let $y_u = y_{\omega_1}^b = y_{\omega_2}^s$ and $y_f = y_{\omega_1}^s = y_{\omega_2}^b$, where subscripts u and f stand for unfavorable and favorable trades for the intermediary. Thus, the optimization problem of the intermediary can be rewritten as

$$\max_{\{y_u, y_f, c\}} \frac{1}{2}\left\{y_u(c - g) + y_f z(c + g)\right\}$$

$$\text{subject to} \quad c \leqslant x + g\left(\frac{y_u - y_f}{y_u + y_f}\right) \quad \text{and} \quad c \leqslant g. \tag{9}$$

It can readily be seen that the monopolist's problem is to balance two forces: on the one hand, she would prefer to avoid the adverse selection cost $(c - g)$ by avoiding all unfavorable bets and accepting all favorable ones; on the other, doing this minimizes the transaction fee that can be charged and jeopardizes the participation of utility traders.

Observe that to set $y_u = y_f = 1$, that is, to accept all bets, yields positive profits as long as $x \geqslant g(1-z)/(1+z)$, which is precisely the condition for the decentralized market to exist. Therefore, if the condition for the existence of a decentralized market is met, the condition for the existence of an intermediated market is also met.

Moreover, even when the above condition is not satisfied, it is possible for the intermediated market to exist. We can verify that when $x = g(1-z)/(1+z)$, we have $\partial E\pi/\partial y_f|_{y_u=y_f=1} < 0$ and $\partial E\pi/\partial y_u|_{y_u=y_f=1} > 0$, meaning that there is a better strategy than accepting all bets in this case. Therefore, the optimal strategy involves positive profits even in cases in which $x < g(1-z)/(1+z)$.

It turns out that:

Theorem 2. *The optimal policy of the informed monopolistic intermediary is given by*

$$(y_u, y_f, c) = \begin{cases} (1, x/(2g-x), g) & \textit{if } x < 2g\frac{1-z}{1+z}, \\ (1, 1, x) & \textit{if } x \geqslant 2g\frac{1-z}{1+z}. \end{cases}$$

Proof. In Appendix A. □

The optimal policy distinguishes between two cases. If x is large enough, then the policy of accepting all bets is optimal. By eliminating the adverse-selection problem for the uninformed, their willingness to pay amounts to the value of gambling, x, which is entirely extracted from them in the form of the transaction fee. The cost to the intermediary of such a policy is trading unfavorably with the informed. However, the trading fee ameliorates this problem: each transaction with an informed party has an expected loss of $(g - x)$.

Observe that if $x \geqslant 2g(1-z)/(1+z)$, then it must be the case $x > g(1-z)/(1+z)$, which is the cutoff point above which this strategy generates positive profits. This strategy then generates positive profits for smaller values of x, yet is not optimal. Indeed, it is dominated by the strategy of accepting all unfavorable bets while accepting a fraction $x/(2g - x)$ of favorable bets. This means that the intermediary offers *favorable*, rather than *adverse*, selection to the uninformed traders. The reason this favors the intermediary is that by doing so she can charge even higher transaction fees, thereby appropriating the surplus $(g - x)$ that the previous strategy left to speculators. In fact, under the previous strategy the transaction fee could not be higher than x, for such a fee would exclude utility traders from the market. The present strategy uses favorable selection to increase the utility traders' willingness to pay, and thereby permit an increase in transaction fees up to the point at which speculators are left without a surplus. All that is given to utility traders in the form of favorable selection is taken back as transaction fees, so that there is no cost to the monopolist. All the extra surplus taken from speculators is an increase in profits. The cost of this strategy, however, is the lower volume of transactions involved in rejecting a fraction of bets, which may overcome the appropriation of speculators' surplus $(g - x)$ if x is large enough.

It is interesting to consider what such a strategy would look like in practice. Think of a broker-dealer who advises her customers on which assets to buy. When she recommends that a client buys asset A and not asset B, truly believing that asset A is undervalued and asset B correctly priced, she is in effect selling favorable selection, improving the expected value of the transaction to her client. Observe that the optimal policy of Theorem 2 considers a mix of true and false (or neutral) recommendations that partially disguise the intermediary's information. The only intention of the policy is to increase the client's willingness to pay up to a point where speculators have no surplus, but no more than that. It is in the monopolist's self-interest to give out some good tips, occasionally, if that increases her client's willingness to pay for her services, because that way she can charge higher fees to speculators too.

In the game of this paper, the equilibrium strategies are known to all. The question arises, however, as to how such a strategy becomes known in practice, and moreover, how the monopolist can commit to its use. This may appear especially troubling when one realizes that from the point of view of an individual utility trader, the intermediary plays a mixed strategy. The answer is connected to the notion of reputation. Even though the present model is static, the equilibrium notion can be justified on the grounds of intertemporal considerations on the part of the intermediary. If she were for instance to "announce" that she will take all bets, and instead takes only those that are favorable to her, her credibility would be lost, and no utility trader would ever trade with her again. If she is patient enough, accepting unfavorable bets would be judged as a good investment in her future credibility. In financial markets, reasoning of this sort seems to be pervasive: a broker-dealer's reputation is essential to stay in business. Successful securities exchanges apparently noticed this early on, putting in practice strict regulations that would ensure investor confidence.

Under the strategy of Theorem 2, the monopolist can keep the market open regardless of the severity of the adverse selection problem. The ability of monopolistic markets to avoid breakdowns, even under large speculative activity, is certainly not a new result. Glosten (1989), for instance, arrives at a similar conclusion, but for a quite different reason. In Glosten's analysis, the monopolist may find it worthwhile to make a negative expected gain trade, because there is a chance that the loss can be recouped later on, in future positive expected gain trades. In effect, Glosten's informationally-disadvantaged market-maker learns some of the information by trading, thereby reducing the informational disadvantage with respect to speculators, while allowing him to make profits on liquidity traders. In this sense, trading today at an expected loss is an investment that may pay off in the form of future trades at a profit. No mechanism of this nature can drive the result here, since the present model is static and the monopolist is informed. Moreover, as the next subsection will show, since the uninformed intermediary does not have the ability of offering favorable selection, she cannot avoid market breakdowns when the adverse-selection problem is too severe.

The ability to keep the market open is not the only difference between these two regimes. There is also a difference in the total number of transactions, and consequently in welfare. These issues are analyzed in Section 6. Before moving into that, however, we consider the cases of an uninformed monopolist and competitive intermediaries.

4.2. Uninformed monopolistic intermediary

The uninformed monopolist differs from the informed one in that she cannot condition her behavior on any informational event. Hence, her problem is:

$$\max_{\{y_u, y_f, c\}} \frac{1}{2}\big\{y_u(c-g) + y_f z(c+g)\big\}$$

$$\text{subject to} \quad c \leqslant x + g\left(\frac{y_u - y_f}{y_u + y_f}\right), \qquad c \leqslant g \quad \text{and} \quad y_u = y_f. \tag{10}$$

Replacing the constraints in the objective leads to the one-variable problem

$$\max_{y_u \in [0,1]} \frac{1}{2}\big((x-g) + z(x+g)\big)y_u \tag{11}$$

with the straightforward solution

$$y_u = y_f = \begin{cases} 1 & \text{if } x \geqslant g\frac{1-z}{1+z}, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

from which it follows that:

Theorem 3. *The market intermediated by an uninformed monopolist is open if and only if the disintermediated market is also open.*

Proof. The condition for trade in Eq. (12) is the same as condition (5b). □

Even though these two regimes do not differ in their ability to avoid market breakdowns, they are not equivalent from a welfare perspective. The centralized market produces more trade. The discussion of this issue takes place in Section 6.

5. Perfectly competitive intermediaries

This section considers the case of a market organized around a set of intermediaries, $j = 1, 2, \ldots, n$, where $n \geqslant 2$. The main thrust is to find out whether such a market will differ from the monopolistic one in the ability to sustain trade under strong adverse selection. A secondary objective is to understand the differences in the distribution of rents between these arrangements. As in the previous section, I distinguish between the cases of informed and uninformed intermediaries.

5.1. Informed intermediaries

The triplet (y_u^j, y_f^j, c^j) is referred to as the contract offered by intermediary j. Let J be the index set for intermediaries, enlarged so as to include an auxiliary intermediary, call it $j = 0$, who "offers" the contract $(y_u^0, y_f^0, c^0) = (0, 0, 0)$. Its inclusion is a devise for guaranteeing that traders' participation constraints are always satisfied.

A utility trader will trade with the intermediary who offers the highest expected utility, that is, with

$$\arg\max_{j\in J} U_{\mathrm{UT}} = \frac{1}{2}(x-g)y_f^j + y_u^j\frac{1}{2}(x+g) - c^j. \tag{13}$$

Similarly, a speculator will trade with the intermediary who offers the maximum payoff

$$\arg\max_{j\in J} U_{\mathrm{S}} = y_u^j\frac{1}{2}\bigl(g - c^j\bigr). \tag{14}$$

If all intermediaries were to offer the same contract, both utility traders and speculators would be indifferent about trading with any of them. It is assumed that in this case, the market is split proportionally among intermediaries, yielding an expected profit of

$$\pi^j = \frac{1}{n}\frac{1}{2}\bigl(y_u(c-g) + y_f z(c+g)\bigr) \tag{15}$$

for each of them, where the j superscripts are dropped without risk of confusion.

Observe that intermediaries and traders of either type have opposite preferences with respect to the contract terms:

$$\frac{\partial U_{\mathrm{UT}}}{\partial y_f} < 0, \qquad \frac{\partial U_{\mathrm{S}}}{\partial y_f} = 0, \qquad \frac{\partial \pi^j}{\partial y_f} > 0; \tag{16a}$$

$$\frac{\partial U_{\mathrm{UT}}}{\partial y_u} > 0, \qquad \frac{\partial U_{\mathrm{S}}}{\partial y_u} \geqslant 0, \qquad \frac{\partial \pi^j}{\partial y_u} < 0; \tag{16b}$$

$$\frac{\partial U_{\mathrm{UT}}}{\partial c} < 0, \qquad \frac{\partial U_{\mathrm{S}}}{\partial c} < 0, \qquad \frac{\partial \pi^j}{\partial c} > 0. \tag{16c}$$

This means that offers that attract traders decrease the intermediaries' per-trade expected profits.

On the other hand, utility traders and speculators have different preferences, which implies that some contracts may attract one group and not the other. If one intermediary offers a contract that attracts speculators and not utility traders, he will lose for sure, and this configuration will not survive in equilibrium. If he offers a contract that only attracts utility traders, then other intermediaries who were making profits will start to experience losses, for they would be left only with speculators. For this reason, equilibrium contracts must be attractive to utility traders.

If competition occurs in contract terms, then a competitive equilibrium requires that no contract can simultaneously yield higher expected profits to a deviant intermediary and higher utility to utility traders. This implies that a competitive equilibrium contract satisfies the following program:

$$\max_{(y_u, y_f, c)} \frac{1}{2}(x-g)y_f + y_u\frac{1}{2}(x+g) - c$$

$$\text{subject to} \quad \frac{1}{n}\frac{1}{2}\bigl(y_u(c-g) + y_f z(c+g)\bigr) = 0. \tag{17}$$

It maximizes the expected utility of utility traders, among those contracts that yield zero profits to the intermediaries.

The zero-profit constraint implies the following relationship between the transaction fee and the probabilities of acceptance:

$$c = \begin{cases} g\frac{y_u - y_f z}{y_u + y_f z} & \text{if } y_u + z y_f \neq 0, \\ 0 & \text{otherwise,} \end{cases} \tag{18}$$

provided that $\min\{y_u, y_f\} > 0$. Replacing it in the objective yields

$$\max_{(y_u, y_f) \in [0,1]^2} V = \begin{cases} \frac{1}{2}(x-g)y_f + y_u \frac{1}{2}(x+g) - g\frac{y_u - y_f z}{y_u + y_f z} & \text{if } \min\{y_u, y_f\} > 0, \\ 0 & \text{if } y_u = y_f = 0. \end{cases} \tag{19}$$

Lemma 4. *The following contract maximizes the utility traders' utility among all pooling contracts—those that attract utility traders and speculators—that yield zero expected profits to intermediaries*:

$$(y_u, y_f, c) = \begin{cases} \left(1, 1, g\frac{1-z}{1+z}\right) & \textit{if } x \geqslant g\frac{1-z}{1+z}, \\ (0, 0, 0) & \textit{otherwise}. \end{cases}$$

Proof. In Appendix A. □

The Lemma 4 contract maximizes utility traders' expected utility and yields zero profits to intermediaries. Hence, it is the only contract that we could expect to see in a pooling competitive equilibrium, that is an equilibrium in which utility traders and speculators participate. Note that this contract makes no use of the intermediaries' information.

We cannot refer to it as an equilibrium contract yet, because we need to verify that there are no separating contracts—contracts that would only attract utility traders—that yield positive profits to a deviant intermediary.

It turns out, however, that those separating contracts exist and, therefore, there is no Nash equilibrium in the present game, in which the choice variables are (y_u^j, y_f^j, c^j). To see this, imagine that all intermediaries are offering the $(1, 1, g(1-z)/(1+z))$ contract and one deviates. She wishes to attract utility traders and not speculators. Observe that by charging a higher transaction fee than her competitors, she will repel speculators (this can be seen in Eq. (13)). Attracting utility traders, however, requires that she decrease the proportion of favorable transaction that she accepts, according to:

$$\begin{aligned} \mathrm{d}U_{\mathrm{UT}} &= \frac{1}{2}(x-g)\mathrm{d}y_f + \mathrm{d}y_u \frac{1}{2}(x+g) - \mathrm{d}c \geqslant 0 \\ \Leftrightarrow \quad \mathrm{d}c &\leqslant \frac{1}{2}(x-g)\mathrm{d}y_f \quad \Leftrightarrow \quad \mathrm{d}y_f \leqslant \frac{-2}{g-x}\mathrm{d}c. \end{aligned} \tag{20}$$

The new profits are higher (that is, positive) under the new contract (y_u', y_f', c') if

$$\begin{aligned} &\frac{1}{2}\left(y_u' z(c'-g) + y_f' z(c'+g)\right) \geqslant 0 \\ \Leftrightarrow \quad & y_f' \geqslant y_u' \frac{g-c'}{g+c'} \quad \Leftrightarrow \quad y_f + \mathrm{d}y_f \geqslant \frac{g-c-\mathrm{d}c}{g+c+\mathrm{d}c} \\ \Leftrightarrow \quad & \mathrm{d}y_f \geqslant -2\frac{c+\mathrm{d}c}{g+c+\mathrm{d}c} \quad \Leftrightarrow \quad \mathrm{d}y_f \geqslant -2\frac{g(1-z)+\mathrm{d}c(1+z)}{2g+\mathrm{d}c(1+z)}. \end{aligned} \tag{21}$$

Observe that z now appears in both terms, not just accompanying the favorable-bet acceptance rate; this is a result of the fact that the strategy selects only utility traders.

Therefore, any increase in fees by, say, $\mathrm{d}c > 0$, together with some favorable selection in the range of

$$-2\frac{c + \mathrm{d}c}{g + c + \mathrm{d}c} \leqslant \mathrm{d}y_f \leqslant -2\frac{\mathrm{d}c}{g - x} \quad \Rightarrow \quad -(1 - z) < \mathrm{d}y_f < 0 \tag{22}$$

(where the last step takes an arbitrarily small increase in fees) will produce profits to the deviant intermediary.

One may be tempted to think that such a deviation will naturally occur. However, the situation that it creates is highly unstable. If the deviant intermediary proceeds along these lines, all her competitors will experience losses, as they would now be trading exclusively with speculators. They will be forced to revise the contracts they offer, catching up with the deviator, and possibly surpassing her. Because there is no Nash equilibrium, if these reactions start, they may not end.

A more plausible outcome is the one selected by the notion of a reactive equilibrium, proposed by Riley (1979). In a reactive equilibrium, each intermediary would refrain from deviating as proposed, because she anticipates that the reaction of her competitors will finally leave her in a worse position (negative profits) than in the current position (zero profits).

Hence, we can state that:

Theorem 5. *The Lemma 4 contract is a competitive (reactive) equilibrium contract.*

Thus, a market organized around a set of perfectly competitive intermediaries is just as bad as the decentralized market with regard to its ability to keep the market open under strong adverse selection. The reason for this is that perfect competition forces expected profits to zero:

$$\frac{1}{n}\frac{1}{2}\big(y_u(c - g) + y_f z(c + g)\big) = 0 \quad \Rightarrow \quad c = \begin{cases} g\,\frac{y_u - y_f z}{y_u + y_f z} & \text{if } y_u + z y_f \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

While the monopolist can set a transaction fee that excludes speculators altogether $(c > g)$, the same cannot be done by competitive intermediaries. The reason is that it would require, if only pooling contracts were possible in equilibrium, that

$$c = g\frac{y_u - y_f z}{y_u + y_f z} > g \quad \Leftrightarrow \quad y_f < 0, \tag{24}$$

which is not feasible. Or, if separating contracts were possible, it would require that

$$c = g\frac{y_u - y_f}{y_u + y_f} > g \quad \Leftrightarrow \quad y_f < 0. \tag{25}$$

This is to say that the zero profit condition, characteristic of perfectly competitive markets, ensures the participation of speculators. The monopolist, on the other hand, can offer favorable selection while at the same time extract the utility trader's surplus, in such a way as to allow a difference between y_u and y_f that does not force her into $y_f < 0$.

5.2. Uninformed intermediaries

Even though the previous section assumed intermediaries were informed, in equilibrium they make no use of their information. The reason is that they could not expect to profit from speculators and using their information against utility traders would decrease their willingness to pay, lowering fees collected from all traders. However, an intermediary who attempts to offer favorable selection may be driven out of business by another who collects a lower fee:

$$\frac{1}{n}\frac{1}{2}\big(y_u(c-g)+y_f(c+g)\big)=0 \quad \Rightarrow \quad c=g\frac{y_u-y_f}{y_u+y_f}\geqslant g, \tag{26}$$

yet

$$c\geqslant g \quad \Leftrightarrow \quad y_f\leqslant 0.$$

In view of this, it is clear that the equilibrium behavior of a set of uninformed, perfectly competitive intermediaries would not differ from the set of informed intermediaries.

6. Discussion

This section compares the three regimes analyzed earlier in terms of welfare and trade.

6.1. Welfare

Considering the partial-equilibrium nature of the model, the proper welfare criterion is total (expected) surplus, as defined by:

$$T=\int\limits_{i\in I} E[u_i]\,\mathrm{d}i+\sum_{j\in J}E[\pi_j] \tag{27a}$$

$$=zE[u_{\mathrm{UT}}]+(1-z)E[u_{\mathrm{S}}]+nE[\pi]. \tag{27b}$$

Since all agents have a reservation utility of zero, their expected utilities coincide with their surpluses. Equation (27b) adds up these surpluses, considering that the mass of utility traders is z, the mass of speculators is $(1-z)$, and that there are n intermediaries making non-negligible transactions.

The total surplus in each regime and its distribution among agents is summarized in the following tables. Table 5 considers the situation in which all three regimes can sustain trade, while Table 6 considers the case where only the monopolistic intermediary is able to keep the market open.

When the market is open under all three regimes, it is necessary to distinguish between two cases:

(1) the one where the monopoly would accept all bets: $x>2g(1-z)/(1+z)$; and
(2) the one in which it would offer favorable selection: $x<2g(1-z)/(1+z)$.

Table 5
Expected utility for each type of agent (assuming $x > g(1-z)/(1+z)$)

	Decentralized	Monopoly		Competition
		$x > 2g(1-z)/(1+z)$	$x < 2g(1-z)/(1+z)$	
Speculator	$zg/2$	$(g-x)/2$	0	$zg/(1+z)$
Utility trader	$(x(1+z)-g(1-z))/2$	0	0	$(x(1+z)-g(1-z))/2$
Intermediary	–	$(x(1+z)-g(1-z))/2$	$xgz/(2g-x)$	0
TOTAL	$zx(1+z)/2$	zx	$zxg/(2g-x)$	zx

Table 6
Expected utility for each type of agent (assuming $x < g(1-z)/(1+z)$)

	Decentralized	Monopoly under $(1, x/(2g-x), g)$	Competition
Speculator	0	0	0
Utility trader	0	0	0
Intermediary	–	$xgz/(2g-x)$	0
TOTAL	0	$zxg/(2g-x)$	0

In case (1), total surplus is the same under monopoly and under perfect competition, the only difference being how much the intermediary is able to obtain from traders. Moreover, total surplus is the same as would be obtained without adverse selection! In other words, in this case private information has a purely redistributive effect, with no consequences for efficiency.

The standard comparison of competitive and monopolistic markets concludes that the latter are inferior from a welfare perspective, because in pursuit of his goal of appropriating more surplus, the monopolist reduces transactions to a socially undesirable level. This does not happen in this model, because it assumes inelastic demand and supply; not only does each trader want to trade a unit amount, but also the populations of each type are homogeneous. Given inelastic demand and supply curves, the only difference in standard analysis between a monopolistic intermediary and no intermediary would be in the distribution of rents, for there is no role for inefficiencies created by unfinished marginal transactions. Of course, allowing for heterogeneity of x among individuals would re-introduce the standard consideration here. This section would then need to re-state the result as follows: "If the inefficiency created by the exclusion of marginal traders does not outweigh the efficiency gains from improving the trading technology with respect to the decentralized market, then [the conclusions above]."

In case (2), in contrast, the monopolistic market underperforms the competitive one, because the policy of offering favorable selection results in a lower volume of transactions. In fact, while accepting all bets yields the highest possible total surplus, that policy involves sharing surplus with speculators, which the monopolist finds suboptimal when that surplus is small. In the process of price discriminating, some volume is lost.

In general, the decentralized market yields a lower total welfare than a centralized one, because some positive surplus transactions are not carried out, in particular those matches in which an uninformed utility trader is matched with an informed speculator, with news adverse to his position. The exception to this rule is the case in which x is not so high as to induce the monopolist to accept all trades, and not so small as to produce a breakdown in the decentralized regime: $g(1-z)/(1+z) < x < 2g(1-z)/(1+z)$. In this situation there are some parameter values ($z \in [-3/4 + \sqrt{17}/4, \sqrt{2} - 1]$) under which the decentralized regime produces more transactions and hence more welfare.

In the case of Table 6, in contrast, the value from gambling is so small ($x < g(1-z)/(1+z)$) that only the monopolist is able to avoid a market breakdown, and consequently, total welfare is higher under this regime than under any other regime. The monopolist, by offering favorable selection, is able to perfectly discriminate among both types of traders, fully appropriating the total surplus.

6.2. *Trading volume*

If the value from trading is too low relative to speculative gains, a market breakdown may occur under a decentralized regime and under a centralized, perfectly-competitive regime as well (Theorems 2 and 5). The conditions for avoiding a market breakdown under each regime are summarized in Table 7.

Conditional on meeting theses conditions, the total volume of trade is obtained from

$$V = \frac{1}{2}\int_{i \in I} E[t_i]\,\mathrm{d}i \tag{28}$$

where t_i is an indicator variable that takes the value 1 if individual i trades at each state, and 0 otherwise. The factor $1/2$ ensures that a purchase and a sale do not count as two transactions. Observe that this equation does not consider trading by intermediaries either, and hence volume in the intermediated regimes is directly comparable to that of the disintermediated regimes. Equation (28) can thus be written as

$$V = \frac{1}{2}\bigl\{zE[t_{\mathrm{UT}}] + (1-z)E[t_{\mathrm{S}}]\bigr\}. \tag{29}$$

In the decentralized economy, on the other hand, utility traders accept in all events, but nevertheless are not always able to trade. Speculators, on the other hand, accept half the time, and a fraction z of those opportunities are able to trade. Hence, volume is given by

$$V = \frac{1}{2}\left(\frac{1}{2}\left(\frac{z}{2}1 + \frac{z}{2}z\right) + \frac{1}{2}\left(\frac{z}{2}1 + \frac{z}{2}z\right) + (1-z)\left(\frac{1}{2}z\right)\right) = \frac{1}{2}z. \tag{30}$$

Observe that this is exactly the volume of trade that would obtain without private information, where only utility traders and all utility traders would trade. However, welfare is lower in the present situation, because some transactions are made for a smaller surplus, and only private gains, as opposed to social gains, are derived from the existence of speculators.

Table 7
Avoiding market breakdown conditions

	Decentralized	Monopolistic	Competitive
Condition	$\frac{x}{g} \geqslant \frac{1-z}{1+z}$	$x \geqslant 0$	$\frac{x}{g} \geqslant \frac{1-z}{1+z}$

Table 8
Expected volume of trade (assuming $x > g(1-z)/(1+z)$)

	Decentralized	Monopoly		Competition
		$x > 2g\frac{1-z}{1+z}$	$x < 2g\frac{1-z}{1+z}$	
Volume	$\frac{1}{2}z$	$\frac{1}{4}(1+z)$	$\frac{1}{4}\frac{2g-x(1-z)}{2g-x}$	$\frac{1}{4}(1+z)$

In centralized regimes in which all intermediaries accept all bets, be they competitive or monopolistic, the volume of trade is augmented to

$$V = \frac{1}{2}\left(z + (1-z)\left(\frac{1}{2}\right)\right) = \frac{1}{4}(1+z). \tag{31}$$

Given that the maximum possible volume that would obtain if all were to accept under every circumstance, is $1/2$, this means that a fraction $(1+z)/2$ of the possible bets actually takes place. The presence of speculators therefore increases volume by $(1-z)/4$. Hence, a fraction $(1-z)/(1+z)$ of transactions is "speculative" and a fraction $2z/(1+z)$ is "fundamental."

In the case of the monopoly that offers favorable selection, total volume is given by

$$V = \frac{1}{2}\left(z\left(\frac{1}{2} + \frac{1}{2}\frac{x}{2g-x}\right) + (1-z)\left(\frac{1}{2}\right)\right) = \frac{1}{4}\frac{2g - x(1-z)}{2g-x}, \tag{32}$$

which is smaller than in the previous case because offering favorable selection requires rejecting some bets.

Table 8 summarizes these results.

7. Further remarks

The opening paragraph of Section 4 suggests a mechanism by which secondary markets become intermediated. There are two crucial aspects in that reasoning. The first is the credibility of the promise of taking all bets, which Section 4.1 has already discussed. The second is that to be viable such a move must generate profits for the intermediary. In the present model, such an opportunity is guaranteed, as Theorem 3 shows.

This paper assumes, however, that intermediation is a costless activity, apart from the adverse selection cost imposed by speculators. Obviously this is unrealistic, and it may well happen that in a given market the adverse selection effect is smaller than the real costs of brokerage, rendering intermediation infeasible.

However, determining whether intermediation is likely or even viable is still a larger task that calls for considering other benefits (possibly of a commercial, as opposed to a financial, nature) associated with brokerage. The present model is only suitable for evaluating the narrow situation in which a secondary market is threatened by the adverse selection effect created by the presence of speculators. In such a situation, intermediation is not only viable but also likely, because of the cascading effect a (credible) intermediary may have on attracting utility traders.

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Appendix A

A.1. Proof of Theorem 2

The informed monopolist solves:

$$\max_{y_u, y_f, c} \frac{1}{2}\big(y_u(c-g) + y_f z(c+g)\big) \tag{A.1a}$$

$$\text{s.t.} \quad c = x + g\left(\frac{y_u - y_f}{y_u + y_f}\right), \tag{A.1b}$$

$$c \leqslant g, \tag{A.1c}$$

$$0 \leqslant y_u, y_f \leqslant 1. \tag{A.1d}$$

The first constraint ensures that utility traders are driven to their reservation utility. The second is the participation constraint for speculators, which can be rewritten as

$$c \leqslant g \quad \Leftrightarrow \quad x + g\frac{y_u - y_f}{y_u + y_f} \leqslant g \quad \Leftrightarrow \quad y_f \geqslant x\frac{y_u}{2g - x}. \tag{A.2}$$

Hence, the problem becomes

$$\begin{aligned}
&\max_{y_u, y_f} \frac{1}{2}\left(y_u\left(x + g\frac{y_u - y_f}{y_u + y_f} - g\right) + y_f z\left(x + g\frac{y_u - y_f}{y_u + y_f} + g\right)\right) \\
&\text{s.t.} \quad y_f \geqslant x\frac{y_u}{2g - x}, \quad 0 \leqslant y_u, y_f \leqslant 1.
\end{aligned} \tag{A.3}$$

Set up the Lagrangean:

$$\begin{aligned}
\max_{y_u, y_f, \lambda} \pounds = {}& \frac{1}{2}\left(y_u\left(x + g\frac{y_u - y_f}{y_u + y_f} - g\right) + y_f z\left(x + g\frac{y_u - y_f}{y_u + y_f} + g\right)\right) \\
&+ \lambda\left(y_f - x\frac{y_u}{2g - x}\right) + \gamma_u(1 - y_u) + \gamma_f(1 - y_f).
\end{aligned} \tag{A.4}$$

The Kuhn–Tucker conditions are:

$$\frac{\partial \pounds}{\partial y_u} = \frac{1}{2}\frac{y_u^2 x + 2y_u y_f x + y_f^2(x - 2g(1-z))}{(y_u + y_f)^2} - \lambda x\frac{y_u}{2g - x} - \gamma_u \leqslant 0,$$

$$\text{wcs } y_u \frac{\partial \pounds}{\partial y_u} = 0, \tag{A.5a}$$

$$\frac{\partial \pounds}{\partial y_f} = \frac{1}{2} \frac{(zx - 2g(1-z))y_u^2 + 2y_u y_f zx + y_f^2 zx}{(y_u + y_f)^2} + \lambda - \gamma_f \leqslant 0,$$

$$\text{wcs } y_f \frac{\partial \pounds}{\partial y_f} = 0, \tag{A.5b}$$

$$\frac{\partial \pounds}{\partial \lambda} = y_f - x \frac{y_u}{2g - x} \geqslant 0, \qquad \text{wcs } \lambda \frac{\partial \pounds}{\partial \lambda} = 0, \tag{A.5c}$$

$$\frac{\partial \pounds}{\partial \gamma_u} = (1 - y_u) \geqslant 0, \qquad \text{wcs } \gamma_u \frac{\partial \pounds}{\partial \gamma_u} = 0, \tag{A.5d}$$

$$\frac{\partial \pounds}{\partial \gamma_f} = (1 - y_f) \geqslant 0, \qquad \text{wcs } \gamma_f \frac{\partial \pounds}{\partial \gamma_f} = 0. \tag{A.5e}$$

The different cases are illustrated in Fig. A.1.

In case A, the monopolist takes all bets. In cases C, D and E speculators are driven to their participation constraints. When this constraint holds, the objective becomes

$$\max_{y_u} \left(\frac{1}{2} x \frac{y_u}{2g - x} z \left(x + g \left(\frac{y_u - x y_u/(2g - x)}{y_u + x y_u/(2g - x)} \right) + g \right) \right). \tag{A.6}$$

Since $\partial/\partial y_u = xzg/(2g - x) > 0$, cases D and E are ruled out. Similarly, if $y_u = 0$, the objective becomes

$$\max_{y_f} \frac{1}{2} \big(y_f z(x) \big). \tag{A.7}$$

Since it is strictly increasing in y_f, cases E and F are ruled out. Case G is a local optimum; however, it is dominated by C (see below).

Now, $y_f = 1$ implies

$$\max_{y_u} \frac{1}{2} \left(y_u \left(x + g \left(\frac{y_u - 1}{y_u + 1} \right) - g \right) + z \left(x + g \left(\frac{y_u - 1}{y_u + 1} \right) + g \right) \right), \tag{A.8}$$

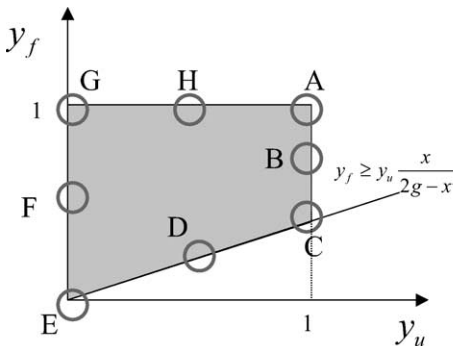

Fig. A.1. The feasible set.

Table A.1
Possible cases

Cases	γ_u	γ_f	λ	
A: $y_u = 1 = y_f > xy_u/(2g - x)$	$+$	$+$	0	To be analyzed
B: $y_u = 1, xy_u/(2g - x) < y_f < 1$	$+$	0	0	To be analyzed
C: $y_u = 1, y_f = xy_u/(2g - x) < 1$	$+$	0	$+$	To be analyzed
D: $y_u < 1, y_f = xy_u/(2g - x) < 1$	0	0	$+$	Ruled out
E: $y_u < 1, y_f = xy_u/(2g - x) < 1$	0	0	$+$	Ruled out
F: $y_u < 1, xy_u/(2g - x) < y_f < 1$	0	0	0	Ruled out
G: $y_u < 1, xy_u/(2g - x) < y_f = 1$	0	$+$	0	To be analyzed
H: $y_u < 1, xy_u/(2g - x) < y_f = 1$	0	$+$	0	Ruled out
I: $0 < y_u, y_f < 1$	0	0	0	To be analyzed

$$
\begin{aligned}
\frac{\partial}{\partial y_u} &= \frac{1}{2}\frac{xy_u^2 + 2xy_u + x - 2g + 2zg}{(y_u + 1)^2} \\
&= \left(y_u + 1 - \sqrt{\frac{(1-z)2g}{x}}\right)\left(y_u + 1 + \sqrt{\frac{2g(1-z)}{x}}\right) \geqslant 0 \\
&\Leftrightarrow \quad y_u \leqslant \sqrt{\frac{(1-z)2g}{x}} - 1.
\end{aligned}
$$

However, in this case the second-order condition (SOC) is not satisfied:

$$
\frac{\partial^2}{\partial y_u^2} = 2g\frac{1-z}{(y_u + 1)^3} > 0. \tag{A.9}
$$

Hence, H is also ruled out (if anything, it would be a local minimum). Table A.1 lists all possible cases.

A.1.1. *Case A*: $y_u = 1 = y_f > x/(2g - x),\ \lambda = 0,\ c = x$

The first-order conditions (FOC) become:

$$
\begin{aligned}
&\frac{\partial \pounds}{\partial y_u} = \frac{1}{2}\frac{x + 2x + (x - 2g(1-z))}{(2)^2} - \gamma_u = 0 \\
&\Rightarrow \quad \gamma_u = \frac{1}{2}x - \frac{1}{4}g(1-z) \geqslant 0 \quad \Leftrightarrow \quad x \geqslant \frac{1}{2}g(1-z), \qquad \text{(A.10)} \\
&\frac{\partial \pounds}{\partial y_f} = \frac{1}{2}\frac{(zx - 2g(1-z)) + 2zx + zx}{(2)^2} - \gamma_f = 0 \\
&\Rightarrow \quad \gamma_f = \frac{1}{2}zx - \frac{1}{4}g + \frac{1}{4}gz \geqslant 0 \quad \Leftrightarrow \quad x \geqslant \frac{1}{2}g\frac{(1-z)}{z}, \\
&\frac{\partial \pounds}{\partial \lambda} = 1 - x\frac{1}{2g - x} \geqslant 0 \quad \Leftrightarrow \quad x \leqslant g. \qquad \text{(A.11)}
\end{aligned}
$$

Evaluating the objective, we can determine where profits are positive:

$$\begin{aligned}
&\pounds = \frac{1}{2}\big((x-g)+z(x+g)\big) = \frac{1}{2}(1+z)x - \frac{1}{2}g(1-z) \geqslant 0 \\
&\quad \Leftrightarrow \quad x \geqslant g\frac{1-z}{1+z}.
\end{aligned} \tag{A.12}$$

A.1.2. Case B: $y_u = 1,\ xy_u/(2g-x) < y_f < 1$

In this case, we are assuming $\gamma_u \geqslant 0,\ \lambda = 0 = \gamma_f$. From the FOC,

$$\begin{aligned}
&\frac{\partial \pounds}{\partial y_f} = \frac{1}{2}\frac{(zx-2g(1-z)) + 2y_f zx + y_f^2 zx}{(1+y_f)^2} = 0 \\
&\quad \propto \quad zx\left(y_f + 1 - \sqrt{\frac{2g(1-z)}{zx}}\right)\left(y_f + 1 + \sqrt{\frac{2g(1-z)}{zx}}\right) = 0 \\
&\quad \Rightarrow \quad y_f = -1 + \sqrt{\frac{2g(1-z)}{zx}} \in [0,1] \quad \Leftrightarrow \quad x \in \left[\frac{1}{2}g\frac{1-z}{z}, 2g\frac{1-z}{z}\right].
\end{aligned} \tag{A.13}$$

However, this point does not satisfy the SOC, which comes from the problem:

$$\begin{aligned}
&\max_{y_f} \pounds = \frac{1}{2}\left(\left(x + g\frac{1-y_f}{1+y_f} - g\right) + y_f z\left(x + g\frac{1-y_f}{1+y_f} + g\right)\right), \\
&\frac{\partial \pounds}{\partial y_f} = \frac{1}{2}\frac{zx - 2g + 2zg + 2zxy_f + y_f^2 zx}{(1+y_f)^2} = 0, \\
&\frac{\partial^2 \pounds}{\partial y_f^2} = 2g\frac{1-z}{(1+y_f)^3} > 0,
\end{aligned} \tag{A.14}$$

so that the above candidate is a local minimum. What we actually have is $\partial \pounds/\partial y_f > 0$ for $y_f \geqslant -1 + \sqrt{2g(1-z)/(zx)}$, and $\partial \pounds/\partial y_f < 0$ otherwise.

A.1.3. Case C: $y_u = 1,\ y_f = x/(2g-x) < 1,\ c = g$

Under the maintained assumptions, the first two FOC's become:

$$\begin{aligned}
&\frac{\partial \pounds}{\partial y_u} = \frac{1}{2}\frac{x + 2x^2/(2g-x) + (x/(2g-x))^2(x-2g(1-z))}{(1+x/(2g-x))^2} - \frac{\lambda x}{2g-x} - \gamma_u = 0, \\
&\frac{\partial \pounds}{\partial y_f} = \frac{1}{2}\frac{(zx-2g(1-z)) + 2xzx/(2g-x) + (x/(2g-x))^2 zx}{(1+x/(2g-x))^2} + \lambda = 0.
\end{aligned}$$

Solving for the Lagrangean multipliers, we obtain

$$\lambda = -\frac{1}{4g}\big(-2zxg + x^2z - 4g^2 + 4gx - x^2 + 4g^2z\big), \qquad \gamma_u = g\frac{x}{2g-x}z.$$

The assumption $\gamma_u \geqslant 0$ is satisfied automatically ($x < g \Rightarrow x < 2g$). On the other hand, $\lambda \geqslant 0$ occurs if

$$\lambda = \frac{1-z}{4g}\left(x - \left(\frac{2-z}{1-z} - \sqrt{\frac{4z-3z^2}{(1-z)^2}}\right)g\right)\left(x - \left(\frac{2-z}{1-z} + \sqrt{\frac{4z-3z^2}{(1-z)^2}}\right)g\right) \geqslant 0$$
$$\text{if} \quad x \leqslant g\left(\frac{2-z}{1-z} - \sqrt{\frac{4z-3z^2}{(1-z)^2}}\right) \quad \text{or} \quad x \geqslant g\left(\frac{2-z}{1-z} + \sqrt{\frac{4z-3z^2}{(1-z)^2}}\right).$$

A.1.4. Case G

The FOC become:

$$\frac{\partial \pounds}{\partial y_u} = \frac{1}{2}\frac{(x - 2g(1-z))}{(1)^2} \leqslant 0, \qquad \frac{\partial \pounds}{\partial y_f} = \frac{1}{2}\frac{zx}{(1)^2} - \gamma_f = 0,$$

which imply

$$\gamma_f = \frac{1}{2}\frac{zx}{(1)^2} \geqslant 0, \qquad \frac{\partial \pounds}{\partial \lambda} = 1 - 0 > 0, \qquad \frac{\partial \pounds}{\partial \gamma_u} = (1-0) \geqslant 0,$$
$$\frac{\partial \pounds}{\partial \gamma_f} = (1-1) = 0.$$

Evaluating the objective function, we have:

$$\pounds_G^* = \frac{1}{2}\left(z\left(x + g\frac{-1}{0+1} + g\right)\right) = \frac{1}{2}xz.$$

A.1.5. Case I

From the FOC, we have:

$$\frac{\partial \pounds}{\partial y_u} = \frac{1}{2}\frac{y_u^2 x + 2y_u y_f x + y_f^2(x - 2g(1-z))}{(y_u + y_f)^2} = 0,$$
$$\frac{\partial \pounds}{\partial y_f} = \frac{1}{2}\frac{(zx - 2g(1-z))y_u^2 + 2y_u y_f zx + y_f^2 zx}{(y_u + y_f)^2} = 0.$$

These equations only admit the solution $y_f = 0 = y_u$, which violates the assumption.

A.1.6. Optimal policy

Profits from the admissible policies are thus given by:

$$\pounds_A^* = \frac{1}{2}\big((x-g) + z(x+g)\big) = \frac{1}{2}(x - g + zx + gz) \quad \left[\geqslant 0 \quad \text{if } x \geqslant g\frac{1-z}{1+z}\right],$$
$$\pounds_C^* = \frac{1}{2}\left(\frac{x}{2g-x}z(g+g)\right) = g\frac{x}{2g-x}z,$$
$$\pounds_G^* = \frac{1}{2}xz.$$

It is important to realize that policies C and G always generate profits. That is, the monopolist always has the chance to profitably keep the market open. However, since

$$\pounds_C^* - \pounds_G^* \geqslant 0 \quad \text{for all } x \geqslant 0,$$

G is never used.

Hence, the optimal policy is obtained from the comparison of $\pounds_A^*$ and $\pounds_C^*$:

$$\pounds_A^* - \pounds_C^* \geqslant 0 \quad \Leftrightarrow \quad x \geqslant 2g\frac{1-z}{1+z}.$$

Therefore, the optimal policy is

$$(y_u, y_f, c) = \begin{cases} (1, x/(2g-x), g) & \text{if } x < 2g\frac{1-z}{1+z}, \\ (1, 1, x) & \text{if } x \geqslant 2g\frac{1-z}{1+z}, \end{cases} \tag{A.15}$$

as stated.

A.2. Proof of Lemma 4

The program to solve in the case of informed competitors is:

$$\max_{(y_u, y_f, c)} \frac{1}{2}(x-g)y_f + y_u\frac{1}{2}(x+g) - c$$
$$\text{subject to} \quad \frac{1}{n}\frac{1}{2}\big(y_u(c-g) + y_f z(c+g)\big) = 0. \tag{A.16}$$

This assumes participation by speculators, which is guaranteed as long as $c \leqslant g$.

The constraint can be written as

$$c = \begin{cases} g\frac{y_u - y_f z}{y_u + y_f z} & \text{if } y_u + zy_f \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{A.17}$$

Participation by speculators can thus be rewritten as

$$g\frac{y_u - y_f z}{y_u + y_f z} \leqslant g \quad \Leftrightarrow \quad y_f \geqslant 0. \tag{A.18}$$

Therefore, participation by speculators is guaranteed, and requires no special constraint in the program.

Replacing c into the objective yields:

$$\max_{(y_u, y_f) \in [0,1]^2} V = \begin{cases} \frac{1}{2}(x-g)y_f + y_u\frac{1}{2}(x+g) - g\frac{y_u - y_f z}{y_u + y_f z} & \text{if } \min\{y_u, y_f\} > 0, \\ 0 & \text{if } y_u = y_f = 0. \end{cases} \tag{A.19}$$

If we treat it as an unconstrained problem, we obtain the following candidate:

$$(y_u, y_f, c)' = \begin{pmatrix} \frac{4gz(g-x)}{(x-g(1+z)/(1-z))^2(1-z)^2} \\ \frac{4gz(g+x)}{(x-g(1+z)/(1-z))^2(1-z)^2} \\ \frac{g(g-x-xz-gz)}{g-x+xz+gz} \end{pmatrix}. \tag{A.20}$$

Table A.2
The boundaries

(y_u, y_f)	c	$V(y_u, y_f)$
$(1,1)$	$g(1-z)/(1+z)$	$x-g(1-z)/(1+z)$
$(0,1)$	$-g$	$x/2+g/2>0$
$(1,0)$	g	$x/2-g/2<0$
$(0,0)$	0	0

This candidate satisfies the FOC, but does not satisfy the SOC:

$$\frac{\partial^2}{\partial y_f^2}(\cdot) = -4gy_u\frac{z^2}{(y_u+y_fz)^3} < 0, \tag{A.21a}$$

$$\frac{\partial^2}{\partial y_u^2}(\cdot) = 4gy_f\frac{z}{(y_u+y_fz)^3} > 0. \tag{A.21b}$$

The point is a saddle.

Observe that this is the only candidate from the FOC. Hence, the global maximum must be attained at the boundary of $[0,1]^2$. If we check the vertices, we get the boundaries as in Table A.2.

Observe also that $c \geqslant 0 \Leftrightarrow g(y_u - y_fz)/(y_u + y_fz) \geqslant 0 \Leftrightarrow y_u \geqslant y_fz$. This means that in $[0,1]^2$ there is a set of contracts for which $c < 0$.

The associated Lagrangean is:

$$\begin{aligned}\max_{(y_u,y_f,\lambda,\gamma)} V = {} & \frac{1}{2}(x-g)y_f + y_u\frac{1}{2}(x+g) - g\frac{y_u - y_fz}{y_u+y_fz} \\ & + \lambda(1-y_f) + \gamma(1-y_u).\end{aligned} \tag{A.22}$$

The Kuhn–Tucker conditions are:

$$\begin{aligned}\frac{\partial V}{\partial y_u} = {} & \frac{1}{2}\frac{1}{(y_u+y_fz)^2}\big(xy_u^2 + 2xy_uy_fz + xy_f^2z^2 + gy_u^2 + 2gy_uy_fz + gy_f^2z^2 \\ & - 4gy_fz - 2\gamma(y_u - y_fz)^2\big) \leqslant 0,\end{aligned} \tag{A.23a}$$

$$\begin{aligned}\frac{\partial V}{\partial y_f} = {} & -\frac{1}{2}\frac{1}{(y_u+y_fz)^2}\big(-xy_u^2 - 2xy_uy_fz - xy_f^2z^2 + gy_u^2 + 2gy_uy_fz + gy_f^2z^2 \\ & - 4gzy_u + 2\lambda(y_u + y_fz)^2\big) \leqslant 0,\end{aligned} \tag{A.23b}$$

$$\frac{\partial V}{\partial \lambda} = 1 - y_f \geqslant 0, \tag{A.23c}$$

$$\frac{\partial V}{\partial \gamma} = 1 - y_u \geqslant 0, \tag{A.23d}$$

with complementary slackness:

$$y_u\frac{\partial V}{\partial y_u} = 0, \qquad y_f\frac{\partial V}{\partial y_f} = 0, \qquad \lambda\frac{\partial V}{\partial \lambda} = 0 \quad \text{and} \qquad \gamma\frac{\partial V}{\partial \gamma} = 0. \tag{A.23e}$$

The following cases analyzed are depicted in Fig. A.2.

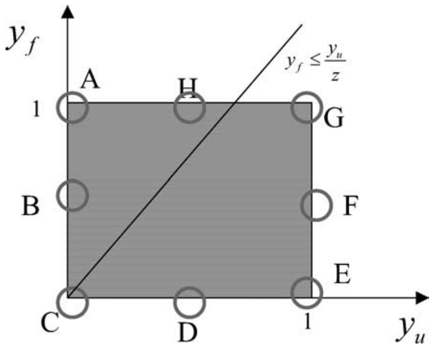

Fig. A.2. The feasible set.

A.2.1. A: $y_u = 0$; $y_f = 1$; $\gamma = 0$; $\lambda \geqslant 0$

Under these assumption, the FOC become:

$$\frac{\partial V}{\partial y_u} = \frac{1}{2}\frac{xz^2 + gz^2 - 4gz}{z^2} \leqslant 0, \tag{A.24}$$

$$\frac{\partial V}{\partial y_f} = -\frac{1}{2}\frac{-xz^2 + gz^2 + 2\lambda z^2}{z^2} = 0, \tag{A.25}$$

which imply, respectively,

$$x \leqslant g\frac{4 - z}{z}, \tag{A.26}$$

$$\lambda = \frac{1}{2}x - \frac{1}{2}g. \tag{A.27}$$

However,

$$\lambda \geqslant 0 \quad \Leftrightarrow \quad x \geqslant g, \tag{A.28}$$

which violates the assumption made throughout the paper, that $x < g$.

A.2.2. B: $y_u = 0$; $y_f < 1$; $\gamma = 0$; $\lambda = 0$

Under these assumption, the FOC become:

$$\frac{\partial V}{\partial y_u} = \frac{1}{2}\frac{xy_f^2z^2 + gy_f^2z^2 - 4gy_fz}{(y_fz)^2} \leqslant 0 \quad \Rightarrow \quad x \leqslant g\frac{4 - y_fz}{y_fz}, \tag{A.29}$$

$$\frac{\partial V}{\partial y_f} = -\frac{1}{2}\frac{-xy_f^2z^2 + gy_f^2z^2}{(y_fz)^2} = \frac{1}{2}x - \frac{1}{2}g = 0. \tag{A.30}$$

The latter condition is impossible under the maintained assumption of $x < g$.

A.2.3. C: $y_u = 0$; $y_f = 0$; $\gamma = 0$; $\lambda = 0$

The objective becomes

$$\max V = \frac{1}{2}(x - g)y_f + y_u\frac{1}{2}(x + g) + \lambda(1 - y_f) + \gamma(1 - y_u),$$

and the associated FOC are:

$$\frac{\partial V}{\partial y_u} = \frac{1}{2}(x+g) \leqslant 0, \tag{A.31}$$

$$\frac{\partial V}{\partial y_f} = \frac{1}{2}(x-g) \leqslant 0, \tag{A.32}$$

the first one of which is clearly impossible. Observe that $V_C = 0$.

A.2.4. D: $0 < y_u < 1;\ y_f = 0;\ \gamma = 0;\ \lambda = 0$

Under these assumption, the FOC become:

$$\frac{\partial V}{\partial y_u} = \frac{1}{2}\frac{xy_u^2 + gy_u^2}{(y_u)^2} = 0 \quad \Leftrightarrow \quad \frac{1}{2}x + \frac{1}{2}g = 0, \tag{A.33}$$

$$\frac{\partial V}{\partial y_f} = -\frac{1}{2}\frac{-xy_u^2 + gy_u^2 - 4gzy_u + 2\lambda y_u^2}{(y_u)^2} \leqslant 0, \tag{A.34}$$

the first of which is clearly impossible.

A.2.5. E: $y_u = 1;\ y_f = 0;\ \gamma \geqslant 0;\ \lambda = 0$

Under these assumption, the FOC become:

$$\frac{\partial V}{\partial y_u} = \frac{1}{2}\frac{x + g - 2\gamma}{(1)^2} = 0 \quad \Rightarrow \quad \gamma = \frac{1}{2}(x+g), \tag{A.35}$$

$$\frac{\partial V}{\partial y_f} = -\frac{1}{2}\frac{-x + g - 4gz}{(1)^2} \leqslant 0 \quad \Rightarrow \quad x \leqslant g(1-4z). \tag{A.36}$$

However, the objective evaluated at this point takes on a negative value:

$$V_E = \frac{1}{2}(x+g) - g = \frac{1}{2}x - \frac{1}{2}g < 0, \tag{A.37}$$

so it is dominated by C, so this cannot be the global maximum.

A.2.6. F: $y_u = 1;\ 0 < y_f < 1;\ \gamma \geqslant 0;\ \lambda = 0$

Under these assumption, the FOC become:

$$\begin{aligned}\frac{\partial V}{\partial y_u} = \frac{1}{2}\frac{1}{(1+y_f z)^2}\big(&x + 2xy_f z + xy_f^2 z^2 + g + 2gy_f z + gy_f^2 z^2 - 4gy_f z - 2\gamma \\ &- 4\gamma y_f z - 2\gamma y_f^2 z^2\big) = 0\end{aligned} \tag{A.38}$$

$$\Rightarrow \quad \gamma = \frac{1}{2}\frac{x + 2xy_f z + xy_f^2 z^2 + g - 2gy_f z + gy_f^2 z^2}{(1+y_f z)^2}, \tag{A.39}$$

$$\frac{\partial V}{\partial y_f} = -\frac{1}{2}\frac{-x - 2xy_f z - xy_f^2 z^2 + g + 2gy_f z + gy_f^2 z^2 - 4gz}{(1+y_f z)^2} = 0, \tag{A.40}$$

from which we get two possibilities. The first one is out of the feasible set:

$$y_f = -\frac{(g-x) + 2\sqrt{gz(g-x)}}{z(g-x)} < 0. \tag{A.41}$$

The second one is not if $x \geqslant g(1-4z)$:

$$y_f = \left(\frac{-(g-x)+2\sqrt{gz(g-x)}}{z(g-x)}\right) \geqslant 0. \tag{A.42}$$

Evaluating,

$$\begin{aligned}
\gamma &= \frac{1}{2}\frac{-x+xz-2\sqrt{gz(g-x)}+g+gz}{z} \geqslant 0 \\
\Leftrightarrow \quad & (1-z)^2\left(x+g\frac{z^2+2z-1-2z\sqrt{2z-1}}{(1-z)^2}\right) \\
& \quad \times \left(x+g\frac{z^2+2z-1+2z\sqrt{2z-1}}{(1-z)^2}\right) \geqslant 0.
\end{aligned} \tag{A.43}$$

Evaluating, we obtain:

$$V_F = \frac{1}{2}\frac{g+3gz-x+xz}{z} - 2g\frac{g-x}{\sqrt{zg(g-x)}}. \tag{A.44}$$

A.2.7. G: $y_u = 1$; $y_f = 1$; $\gamma \geqslant 0$; $\lambda \geqslant 0$

Under these assumption, the FOC become:

$$\begin{aligned}
\frac{\partial V}{\partial y_u} &= \frac{1}{2}\frac{x+2xz+xz^2+g+2gz+gz^2-4gz-2\gamma-4\gamma z-2\gamma z^2}{(1+z)^2} = 0 \\
\Rightarrow \quad & \gamma = \frac{1}{2}\frac{x+2xz+xz^2+g-2gz+gz^2}{(1+z)^2} \geqslant 0 \\
\Leftrightarrow \quad & -g\frac{(1-z)^2}{(1+z)^2} \leqslant x,
\end{aligned} \tag{A.45}$$

$$\begin{aligned}
\frac{\partial V}{\partial y_f} &= -\frac{1}{2}\frac{-x-2xz-xz^2+g+2gz+gz^2-4gz+2\lambda+4\lambda z+2\lambda z^2}{(1+z)^2} = 0 \\
\Rightarrow \quad & \lambda = \frac{1}{2}\frac{x+2xz+xz^2-g+2gz-gz^2}{(1+z)^2} \geqslant 0 \\
\Leftrightarrow \quad & x \geqslant g\frac{(1-z)^2}{(1+z)^2}.
\end{aligned} \tag{A.46}$$

Therefore, this is a local maximum if

$$x \geqslant g\frac{(1-z)^2}{(1+z)^2}, \tag{A.47}$$

in which case

$$V_G = \frac{1}{2}(x-g) + \frac{1}{2}(x+g) - g\frac{1-z}{1+z} = x - g\frac{1-z}{1+z}. \tag{A.48}$$

Observe that, given that $z \in (0,1)$,

$$\frac{(1-z)^2}{(1+z)^2} < \frac{1-z}{1+z} \tag{A.49a}$$

$$\Leftrightarrow \quad g\frac{(1-z)^2}{(1+z)^2} < g\frac{1-z}{1+z} \tag{A.49b}$$

$$\Leftrightarrow \quad x - g\frac{(1-z)^2}{(1+z)^2} > x - g\frac{1-z}{1+z}. \tag{A.49c}$$

A.2.8. H: $0 < y_u < 1$; $y_f = 1$; $\gamma = 0$; $\lambda \geqslant 0$

Under these assumption, the FOC become:

$$\begin{aligned}
\frac{\partial V}{\partial y_u} &= \frac{1}{2}\frac{xy_u^2 + 2xy_uz + xz^2 + gy_u^2 + 2gy_uz + gz^2 - 4gz}{(y_u+z)^2} = 0,\\
\frac{\partial V}{\partial y_f} &= -\frac{1}{2}\frac{1}{(y_u+z)^2}\big(-xy_u^2 - 2xy_uz - xz^2 + gy_u^2 + 2gy_uz + gz^2 - 4gzy_u + 2\lambda y_u^2\\
&\qquad + 4\lambda y_uz + 2\lambda z^2\big) = 0
\end{aligned} \tag{A.50}$$

from where we obtain:

$$y_u = \begin{cases} -\dfrac{1}{2(x+g)}\big(2gz + 2xz + 4\sqrt{(xgz + g^2z)}\big) < 0,\\[2mm] \dfrac{1}{x+g}\big(-gz - xz + 2\sqrt{gz(x+g)}\big), \end{cases} \tag{A.51}$$

$$\lambda = \frac{1}{2}\frac{xy_u^2 + 2xy_uz + xz^2 - gy_u^2 + 2gy_uz - gz^2}{(y_u+z)^2}, \tag{A.52}$$

respectively. Note that

$$y_u \geqslant 0 \quad \Leftrightarrow \quad z^2(x+g)\left(x - g\frac{4-z}{z}\right) \leqslant 0 \quad \Leftrightarrow \quad x \leqslant g\frac{4-z}{z} \tag{A.53}$$

and that

$$\begin{aligned}
&\lambda \geqslant 0\\
&\quad\Leftrightarrow \quad (1-z)^2\left(x - \frac{2z + 1 - z^2 + 2\sqrt{z(2-z)}}{(1-z)^2}g\right)\\
&\qquad\qquad \times \left(x - g\frac{2z + 1 - z^2 - 2\sqrt{z(2-z)}}{(1-z)^2}\right) \leqslant 0,\\
&\quad\Rightarrow \quad g\frac{2z + 1 - z^2 - 2\sqrt{z(2-z)}}{(1-z)^2} \leqslant x \leqslant g\frac{2z + 1 - z^2 + 2\sqrt{z(2-z)}}{(1-z)^2}.
\end{aligned} \tag{A.54}$$

In this case,

$$c = -\big(gz + xz - \sqrt{gz(x+g)}\big)\frac{g}{\sqrt{gz(x+g)}} \tag{A.55}$$

and the value of the objective function becomes:

$$V_H = 2gz\frac{x+4g}{\sqrt{gz(x+g)}} + \frac{1}{2}(x - 3g - gz - xz). \tag{A.56}$$

A.2.9. Global maximum

We are left with three possibilities:

$$V_F = \frac{1}{2}\frac{g+3gz-x+xz}{z} - 2g\frac{g-x}{\sqrt{zg(g-x)}}, \tag{A.57}$$

$$V_G = x - g\frac{1-z}{1+z}, \tag{A.58}$$

$$V_H = 2gz\frac{x+4g}{\sqrt{gz(x+g)}} + \frac{1}{2}(x-3g-gz-xz). \tag{A.59}$$

Now, observe that $V_F \leqslant V_G$:

$$\frac{1}{2}\frac{g-x+3gz+xz}{z} - 2g\frac{g-x}{\sqrt{zg(g-x)}} \leqslant x - g\frac{1-z}{1+z}$$

$$\Leftrightarrow \quad (x-g)\left(x-\left(z^2-2z+1\right)\frac{g}{1+2z+z^2}\right)^2 \leqslant 0, \tag{A.60}$$

which is always true by assumption.

On the other hand, $V_G \geqslant V_H$ if

$$(1+z)^4\left(x+g\frac{(1-z)^2}{(1+z)^2}\right)^2 \geqslant 0, \tag{A.61}$$

which, again, is always true.

Therefore, the unique maximum is attained by the strategy $(1, 1, g(1-z)/(1+z))$, that yields a positive profit as long as

$$x \geqslant g\frac{1-z}{1+z}. \tag{A.62}$$

If this condition is not met, the best contract is $(0, 0, 0)$, as stated.

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